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Investigating the Convergence of Some Selected Properties on Block Predictor-Corrector Methods and it's Applications

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Abstract: Investigating the convergence of some selected properties on block predictor-corrector methods and its implementation will be considered. This investigation will provide some justification and theorems that guarantee the convergence of the method. Some aspects of the block predictor-corrector methods to be investigated includes order, convergence and implementation. However, predictor-corrector methods attracts a lot of computational benefits which guarantees step size variation, convergence criteria (stopping criteria) and minimizing error. Again, existence and uniqueness of the method will be recognized. Implementation of this approach will depend on the principal local truncation error on a pair of predictor-corrector method of Adams type formulas either in $P(EC)^m$ or $P(EC)^m$ mode.

Key words: Predictor-corrector methods order, convergence, principal local truncation error, stopping criteria, implementation

INTRODUCTION

According to Gear (1971), the convergence of predictor-corrector methods was inquired in the prestigious article of Mises, this was accompanied by an overwhelming number of papers amending the error bounds and enforcing the concept to other autonomous multistep methods. Generally, convergence proof for differential equations, not withstanding was first started by Dahlquist (1956) who established necessary and sufficient conditions for convergence. Also, major significance was presented in the proofs by the ideas of Butcher, implies that multistep expressions are composed as single-single step expressions in an advanced multidimensional space. Moreover, for sensible convergence of computational methods, the differential equation problem:

$$y\,(x) = f\,(x\,,\,y),\ y(a) = \alpha,\ x\in[a,\,b] \text{ and } f\!: R\times R^m\rightarrow R^m \quad (1)$$

must possess a unique solution. Consequently, we arrive by adopting the assumptions stated below. The solution to Eq. 1 is generally written as:

$$\sum\nolimits_{i=1}^{j}\alpha_i y_{n+i} = h\sum\nolimits_{i=1}^{j}\beta_i f_{n+1} \quad (2)$$

where, the step size is h, $\alpha_j = 1$, α_i, $i = 1, ..., j$, β_j are unknown constants which are uniquely specified such that the formula is of order j as discussed by Akinfenwa *et al*. (2013). We assume that $f\in R$ is sufficiently differentiable on $x\in[a, b]$ and satisfies a global Lipchitz condition, i.e., there is a constant $L\geq 0$ such that:

$$\left|f(x,\,y)-f\left(x,\,\bar{y}\right)'\right|\leq L\left|y-\bar{y}\right|,\ \forall y,\ \bar{y}\in R$$

Under this presumptuousness (Eq. 1) assured the existence and uniqueness defined on $x\in[a, b]$ as well as viewed to fulfill the Weierstrass theorem, for example (Gear, 1971; Lambert, 1973; Xie and Tian, 2014) for details. Where a and b are finite and $y^{(l)}$ $[y_1^{(i)}, y^{(l)}_2, ..., y^{(l)}_n]$ for i = 0 (1)3 and $f = [f_1, f_1, ..., f]^T$, originate in real life applications for problems in science and engineering such as fluid dynamics and motion of rocket as presented by Mehrkanoon *et al*. (2010).

According to Hairer *et al*. (1987), the convergence of variable step size Adams methods was considered by Piotrowsky and Stability. To establish convergence of the general case, the vector $y_n = (y_{n+k}, ..., y_{n+k}, y_n)$ was presented. In proportionality to:

$$Y_{i+1} = (A\otimes I)Y_i + h\Phi(x_i,\,Y_i,\,h)\ i\geq 0$$

the method:

$$y_{n+k}+\sum\nolimits_{j=1}^{k-1}\alpha_{jn}y_{n+j} = h_{n+k-1}\sum\nolimits_{j=0}^{k}\beta_{jn}f_{n+j}$$

then certainly becomes equivalent to:

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$$Y_{i+1} = (A \otimes I)Y_n + h_{n+k-1}\phi_n(x_n, Y_n, h_n)$$

where, A_n is referred to as the companion matrix and:

$$\phi_n(x_n, Y_n, h_n) = (e_1 \otimes I)\psi_n\phi_n(x_n, Y_n, h_n)$$

The value $\psi = \psi_n$ (x_n, y_n, h_n) is implicitly specified as:

$$\psi = \sum_{j=0}^{k-1} \beta_{jn} f(x_{n+j}, y_{n+j}) + \beta_{kn} f(x_{n+k}, h\psi - \sum_{j=0}^{k-1} \alpha_{jn} y_{n+j}$$

where the coefficients α_{jn} and β_{kn} are in actual fact depends on the ratios $w_i = h_i/h_{i-1}$ for:

$$i = n+1, ..., n+k-1$$

Furthermore by letting:

$$Y(x_n) = (y(x_{n+k-1}), ..., y(x_{n+1}), y(x_n))^T$$

the exact values can be approximated by y_n. The convergence theorem can now be phrased as accordingly. Afterward, the idea of convergence shows the attribute that by making use of a sufficiently small step size and precise calculation, the numeric resolution can be formed arbitrary close to the exact resolution. In a multivalue method, there is an initial parameter α_0 or y_0 which sometimes may not become wholly defined by the starting parameters. For instance, a two-step scheme of a first order equation, given y_0 but in addition require to recognize y_1. Subsequently, one or some computational approach will definitely initiate errors into y_1 and perhaps as well y_0, hence, it is very important to permits these in the explanation of convergence whenever if it must be pragmatic. Gear (1971) for details.

Definition (convergence): The multistep method (Eq. 2) is said to be convergent, if for initial value problems (Eq. 1) meets the requirement of unique solution stated above:

$$y(x) - y_h(x) \to 0 \text{ for } h \to 0, x \in [x_0, \bar{x}]$$

if the initial values satisfy (Gear, 1971):

$$y(x_0 + ih) - y_h(x_0 + ih) \to 0 \text{ for } h \to 0, 1, ..., k-1$$

Definition: A multistep (multivalue) method for first order equations is convergent if, for any differential equation satisfying a Lipschitz condition, the computed solution $y_n[\alpha_n]$ converges to y(x) [a(x)] uniformly in $0 \leq x \leq b$ as $y_0 \to y(0)$ $[a_0 \to a(0)]$ and $n \to \infty$ with h = x/n (Gear, 1971).

Definition: A multivalue method is q-convergent for pth order equations if, for any pth order q-differential equation satisfying a Lipschitz condition, the computed solution is α_n such that:

$$\|\alpha_0 - \alpha(x)\|_h^{p-q+1} \to 0$$

uniformly for $0 \leq x \leq$ b as:

$$\|\alpha_0 - \alpha(x)\|_h^{p-q+1} \to 0$$

and $n \to \infty$ with h = x/n. This is called a 1-convergent method simply referred to as convergent (Gear, 1971). The next definitions and theorems guarantees the convergence of Eq. 2.

A multistep (multivalue) method is stable for first order equations if, for any first order equation satisfying a Lipschitz condition, there exist constants K and h_0 such that:

$$\|y_n - y_n^*\| \leq K\|y_0 - y_0^*\|$$
$$\|\alpha_n - \alpha_n^*\| \leq K\|\alpha_0 - \alpha_0^*\|$$

for all $0 \leq x \leq b$ and all h = (x/n)$\in$(0, h_0) where y_n and $y_n^*[\alpha_n, a_n^*]$ are two numerical solutions (Gear, 1971).

Theorem assume that:

$$y_{n+k} + \sum_{j=1}^{k-1} \alpha_{jn} y_{n+j} = h_{n+k-1} \sum_{j=0}^{k} \beta_{jn} f_{n+j}$$

is stable of order p and has bounded coefficients α_{jn} and β_{jn}. The starting values satisfy:

$$\|Y(x_0) - Y_0\| = O(h_0^p)$$

The step size ratios are bounded ($h_i/h_{i-1} \leq \Omega$). Thus, the method is convergent of order p, i.e., for each differential equation y(x) = f(x, y), y(α) = α with f sufficiently differentiable the global error satisfies:

$$\|Y(x_0) - Y_0\| = Ch^p \text{ for } x_n \leq \bar{x}$$

where, h = max h_j (Hairer *et al*., 1987). Again we justifies the essence of convergence on multistep methods using the following definitions and theorems.

Theorem 2: If the multistep method (Eq. 2) is convergent, then it is necessarily stable and consistent (i.e., of order 1: ρ (1) = 0, ρ (1) = σ (1) as seen by Hairer *et al*. (1987). Thus, from the above definitions and theorems, investigating the convergence of some selected properties on block predictor-corrector method demands that some criteria must be satisfied to guarantee the implementation of this method as sited by Gear (1971) and Hairer *et al*. (1987). Therefore, the main objective of this paper will be to investigate the convergence of some selected properties on block predictor-corrector methods otherwise known as $P(EC)^m$ or $P(EC)^m$ mode for solving ODEs.

Order of block predictor-corrector method definition 4: If the L_h operator is defined by:

$$L_h(y(x)) = \sum_{i=0}^{k}\left(\alpha_i^{y(x-ih)} + \frac{h^p}{p!}\beta_i y^{(p)}(x-ih)\right)$$

then the order is the largest integer such that:

$$L_h(y(x)) = 0\left(h^{r+1}\right)$$

whenever, $y \in C_{r+1}$. If we assume that $y \in C_{r+2}$, we can substitute Taylor's series with remainder terms of $0\ (h^{r+1})$ for y (x-ih) and $h^q y^{(q)}$ (x-ih) to get:

$$L_h(y(x)) = \sum_{q=0}^{r+1} C_q h^q y^{(q)}(x) + 0(h^{r+2})$$

Where:

$$C_q = \begin{cases} \displaystyle\sum_{i=0}^{k}\frac{(-i)^q}{q!}\alpha_i,\ q < p \\ \left[\displaystyle\sum_{i=0}^{k}\frac{(-i)^q}{q!}\alpha_i + \frac{(-i)^{q-p}}{p!(q-p)!}\beta_i\right],\ r+1 \ge q \ge p \end{cases}$$

This demonstrates that the order is checked by the coefficients of the method. If we define the polynomials p and σ as earlier, it is observe that:

$$C_q = \begin{cases} \dfrac{1}{q!}\left[\left(\xi\dfrac{d}{d\xi}\right)^q\left(\xi^{-k}\sigma(\xi)\right)\right]_{\xi=1} \\ \dfrac{1}{q!}\left[\left(\xi\dfrac{d}{d\xi}\right)^q\left(\xi^{-k}\rho(\xi)\right) + \begin{pmatrix} q \\ p \end{pmatrix}\left(\xi\dfrac{d}{d\xi}\right)^{q-p}\left(\xi^{-k}\rho(\xi)\right)\right]_{\xi=1} \end{cases},\ q \ge p$$

From the above, it complies that if the order $r \ge p$:

$$\begin{aligned} &\rho(1) = 0 \\ &\rho'(1) = 0 \\ &\dots \\ &\rho^{(p-1)}(1) = 0 \\ &\rho^{(p)}\ \sigma(1) = 0 \end{aligned} \tag{3}$$

In converse manner, if Eq. 3 agrees, the order is $\ge p$ afterward $C_q = 0$ for $q \le p$. It is notice that this specifies the order of a method established on a corrector exclusively that is one in which the formula is explicit or the corrector is reiterated to convergence. Thus, the order of a multivalue method or a predictor-corrector multistep method obviously has to be determined in terms of the result of a differential equation. This can be specify in accordance with Gear (1971).

Definition 5: If **a** (x) is the correct value of the vector **a** for some h at time x and then define:

$$\begin{aligned} \bar{a}_{(0)} &= Aa(x-h) \\ \bar{a}_{(m+1)} &= \bar{a}_{(m)} + IF(\bar{a}_{(m)}) \\ \bar{a}(x) &= \bar{a}_{(M)} \end{aligned}$$

then the order of the method pth order equations is the largest r such that if F represents any differential equation of order p with a solution C_{r+1}:

$$\bar{a}_{(x)} - a(x) = 0(h^{r+1})$$

Thus, it is seen that the order of the corrector is greater the order of the predictor by unity for each one corrector iteration up to the order of the corrector. Nevertheless, suppose a predictor-corrector method can be showed as a multivalue method (as it can be if p = 1, assume f rely y on and x only or whenever bounds on the predictors for y, ..., $y^{(p-1)}$ are met), the order of the predictor can be described from the form of the matrix A if the scheme is set in the normal form. Suppose the predictor-corrector method possesses order r, then the first r+1 columns of A take the form:

$$\begin{bmatrix} 1 & 1 & . & . & . & & 1 \\ & 1 & . & . & . & & r' \\ & & & & & & \\ & & & & & & r' \\ & 0 & & & & & \\ & & & & & & 1 \\ . & . & . & . & . & . & . \\ & & & 0 & & & \end{bmatrix}$$

Thus, indicates that, the Pascal triangle matrix in the upper section and zeros in the lower section will be form Gear (1971).

Theorem 3: If the multistep method:

$$\sum_{i=0}^{k}\left(\alpha_i y_{n-i} + \beta_i \frac{h^p}{p_l} f_{n-1}\right) = 0$$

is convergent for order equations, then the order of:

$$\sum_{i=0}^{k}\left(\alpha_i y_{n-i} + \beta_i \frac{h^p}{p_l!} f_{n-i}\right) = 0$$

is at least p (Gear, 1971).

Theorem 4: The order of a predictor-corrector method for first order equations must be greater than or equal to one if it is convergent.

Proof: Suppose the polynomials specifying the predictor and corrector method be p, σ and p*, σ*, severally.

Theorem 3: Proved that the order of the corrector must be at least unity, the solely case to look at is a predictor order of minus one and a single correction of order one or greater. Nevertheless, the is normalize so that $\alpha_0 = \alpha_0 = -1$. From the explanation of order presented by Definition 5: gives:

$$\begin{aligned}&\sum_{i-0}^{k}\left[\alpha_i^* y(x-ih) + h\beta_0^* y(x-ih)\right] + \\ &h\beta_0^* f\left[\sum\left[\alpha_i y(x-ih) + h\beta_i y(x-ih)\right]\right] - y(x) = 0\left(h^{r+1}\right)\end{aligned} \quad (4)$$

for any differential equation $Y = f(x, y)$ whose solution $Y(x) \in C_{r+1}$. Considering the first order equation for this arrives at:

$$e^{x-kh}\left\{\rho^*\left(e^h\right) + h\sigma^*\left(e^h\right) + h\beta_0^*\left[\rho\left(e^h\right) + h\sigma\left(\varepsilon^h\right)\right]\right\} = 0\left(h^{r+1}\right) \quad (5)$$

Assume ξ is a root of:

$$\rho^*(\xi) + h\sigma^*(\xi) + h\beta_0^*\rho(\xi) + h^2\beta_0^*\sigma(\xi) = 0 \quad (6)$$

then $y_n = \xi^n$ is a solvent of the predictor-corrector method. Studying (Eq. 6) for an answer of this type $\xi = e^h + \Delta$ where, Δ is lowly. Thus, obtain by replacement:

$$\begin{aligned}&\rho_\Delta^{*'}\left(e^h\right) + \rho^*\left(e^h\right) + h\sigma^*\left(e^h\right) + \\ &h\beta_0^*\rho\left(e^h\right) + h^2\beta_0^*\sigma\left(e^h\right) = 0\left(\Delta^2 + h\Delta\right)\end{aligned} \quad (7)$$

The final four terms of the left-hand side are $0(h^{r+1})$ by Eq. 6 and $\rho^{*'}(e^h)+\rho^{*'}(1)+0(h)$ thus:

$$\Delta = \frac{1}{\rho^{*'}(1)} 0\left(\Delta^2 + h\Delta + h^{r+1}\right)$$

Through Theorem 3 the corrector possesses order 1, thence $p^*(1) = 0$. Subsequently, the method converges, it meets the root condition that “A Q-convergent Multivalue Method satisfies the q-root condition” and so $p^*(\xi)$ does not possess a double root at $\xi = 1$. Therefore, $P^*(1) \neq 0$. Consequently:

$$\Delta = Kh^{r+1} + 0\left(h^{r+2}\right) \quad k \neq 0$$

Suppose:

$$\begin{aligned}y_n = \xi^n &= \left(e^h + Kh + 0\left(h^2\right)\right)^n \\ &= e^{(K+1)t} + Kh + 0(h)^2\end{aligned}$$

where, $x = nh$. Finally, given the initial value problem $y' = y$, together with initial condition $y(0) = 1$ and initiating conditions $y_I = (e^h + \Delta)$, $0 \le i \le k$, converges to the answer e^t assuming the order of the predictor is minus one as required (Gear, 1971).

CONVERGENCE OF BLOCK PREDICTOR-CORRECTOR METHOD

Assume P specifies the practical application of the block predictor, C a block corrector practical application, $y_{n+k}^{(0)}$ with E as the evaluation practical application of f with respect to given numeric values of its parameter. Let be computed from the block predictor:

$$f_{n+k}^{(0)} \equiv f(X_{n+k}, y_{n+k}^{(0)})$$

is computed one time and employ the corrector at one time as well to get; $y_{n+k}^{(1)}$ this describe the computing as PEC. Further, appraisal of:

$$f_{n+k}^{(1)} \equiv f(X_{n+k}, y_{n+k}^{(1)})$$

succeeded by another practical application of the corrector gives $y_{n+k}^{(2)}$ and thus, denoted by $PEC^{(2)}$. Implementing the practical application of the block corrector many times can be described as $PEC^{(m)}$. Since, m is constant, $y_{n+k}^{(m)}$ is accepted as the computational solution at X_{n+k}. At this point, the last computational value for f_{n+k} is preferred as:

$$f_{n+k}^{(m-1)} \equiv f(X_{n+k}, y_{n+k}^{(m-1)})$$

and this will foster whether or not to execute. Suppose this execution is concluded, the mode is denoted by $P(EC)^m$ or $P(EC)^m E$. Eventually, the decision clearly moves the next step of the execution, when both predicted and corrected numerical values for Y_{n+k+1} will rely on whether f_{n+k} is accepted as $f_{n+k}^{(m)}$ or $f_{n+k}^{(m-1)}$. Finally, for a given m, $P(EC)^m$ or $P(EC)^m E$ mode utilize the corrector the same number of times; only $P(EC)^m E$ requires one more evaluation per step than $P(EC)^m$ as discussed by Lambert (1973, 1991). The justification of block predictor-corrector method can be established with the following definition and theorems.

Definition 6: A method for a pth order equation is called consistent if its order is at least p. The effect that is plausibly true is that q-stability and consistency are necessity and sufficient conditions for q-convergence of the differential equation (Gear, 1971).

Theorem 4: A stable consistent multivalue method for first order equations is converge-nt (Gear, 1971).

Theorem 5: If a multivalue method for first order equations has order r and if the starting errors e_0 are bounded by $D'h^{r'}$, the error at time x = Nh is bounded by:

$$\|e_N\| \le \frac{Dh^r}{C_0}\left(e^{c_0 c_1^{x-1}}\right) + D'h^{r'} c_1 e^{c_0} c_1^x \tag{8}$$

if the solution. $e^{(x)'} \epsilon C_{r+1}$. Lipschitz condition on y in that region, a necessary and sufficient condition for convergence is that:

$$\psi(y(x), x, 0) = f(y(x), x) \tag{9}$$

Equation 10 is called the condition of consistency. Since, by suitable choice of initial conditions, Y (x) can take on any value for a given, x (Eq. 10) will hold for any y in the form (Gear, 1971):

$$\psi(y, x, 0) = f(y, x)$$

Theorem 6: If ψ (y, x, h) satisfies a Lipschitz condition in L, then the method given by one step method is stable (Gear, 1971).

Theorem 7: Let $\{y^{[m]}{}_{n+1}\}$ be a sequence of approximations of y_{n+1} obtained by a PECE... method. If:

$$\left|\frac{\partial f}{\partial y}(x_{n+1}, y)\right| \le L$$

(for all y near y_{n+1} including $y_{n+1}^{(0)}, y_{n+1}^{(1)} \ldots$) where L satisfies the condition $L<1/|h\beta_0|$, then the sequence $\{y_{n+1}^{[m]}\}$ converges to y_{n+1}.

Proof: The numeric solution satisfies the equation:

$$y_{n+1} = \sum_{i=0}^{j-1} \alpha_i y_{n+i} + h\beta_0\, f\left(x_{n+j}, y_{n+1}\right) + h\sum_{i=0}^{j-1} \beta_i f_{n+i}$$

The corrector satisfies the equation:

$$y_{n+1}^{(m+1)} = \sum_{i=0}^{j-1} \alpha_i y_{n+i} + h\beta_0\, f\left(x_{n+j}, y_{n+1}\right) + h\sum_{i=0}^{j-1} \beta_i f_{n+i}$$

Subtracting these two equations, we obtain:

$$y_{n+1} - y_{n+1}^{(m+1)} = h\beta_0\left[\left| f\left(x_{n+1}, y_{n+1}\right) - f\left(x_{n+1}, y_{n+1}^{(m)}\right)\right|\right]$$

Applying the Lagrange mean value theorem to arrive at:

$$y_{n+1} - y_{n+1}^{(m+1)} = h\beta_0\left(y_{n+1} - y_{n+1}^{(m)}\right)\frac{\partial f}{\partial y}\left(x_{n+1}, y^*\right)$$

where, $y_{n+1}^{(m)} \le y^* \le y_{n+1}$. Thus:

$$\begin{aligned}\left|y_{n+1} - y_{n+1}^{(m+1)}\right| &\le |h\beta_0|\left|y_{n+1} - y_{n+1}^{(m)}\right|\left|\frac{\partial f}{\partial y}\left(x_{n+1,y}\right)\right| \\ &\le hL|\beta_0|\left|y_{n+1} - y_{n+1}^{(m)}\right| \\ &\le \left[hL|\beta_0|\right]^m \left|y_{n+1} - y_{n+1}^{(0)}\right|\end{aligned}$$

Now:

$$\lim\left|y_{n+1} - y_{n+1}^{(m+1)}\right| \to 0$$

If:

$$hL|\beta_0| < 1 \text{ or } L < \frac{1}{h|\beta_0|}$$

This means that the conclusion of Theorem holds as seen by Jain *et al.* (2007).

IMPLEMENTATION OF BLOCK PREDICTOR-CORRECTOR METHOD

Holding to Jain *et al.* (2007) and Lambert (1973, 1991), the implementation in the $P(EC)^m$ or $P(EC)^m E$ mode becomes significant for the predictor-corrector method, if both are individually of like order and this requirement might makes it necessary for the stepnumber of the predictor method to be one step higher than that of the corrector method. Therefore, the mode $P(EC)^m$ or $P(EC)^m E$ can be otherwise be called predictor-corrector method which is now formally examined as follows for m = 1, 2, ...: $P(EC)^m$:

$$y_{n+j}^{[0]} + \sum_{i=0}^{j-1} \alpha_i^{\bullet} y_{n+j}^{[m]} = h \sum_{i=0}^{j-1} \beta_i^{\bullet} f_{n+1}^{[m]},\ f_{n+j}^{[s]} \equiv f(x_{n+j}, y^{[s]})\Big\},$$
$$y_{n+j}^{[s+1]} + \sum_{i=0}^{j-1} \alpha_i y_{n+j}^{[m]} = h\beta_j f_{n+j}^{[s]} + h \sum_{i=0}^{j-1} \beta_i f_{n+j}^{[m]},\ s = 0, 1, ..., m-1 \tag{10}$$

$P(EC)^m E$:

$$y_{n+j}^{[0]} + \sum_{i=0}^{j-1} \alpha_i^{\bullet} y_{n+j}^{[m]} = h \sum_{i=0}^{j-1} \beta_i^{\bullet} f_{n+1}^{[m]},\ f_{n+j}^{[s]} \equiv f(x_{n+j}, y^{[s]})\Big\},$$
$$y_{n+j}^{[s+1]} + \sum_{i=0}^{j-1} \alpha_i y_{n+j}^{[m]} = h\beta_j f_{n+j}^{[s]} + h \sum_{i=0}^{j-1} \beta_i f_{n+j}^{[m]},$$
$$s = 0, 1, ..., m-1\ f_{n+j}^{[m]} \equiv f\left(x_{n+j}, y_{n+j}^{[m]}\right)$$

Noting that as $m \to \infty$ the result of evaluating with either of the above mode will slope to those given by the mode of correcting to convergence. Moreover, predictor and corrector pair based on Eq. 1 can be applied. The mode $P(EC)^m$ or $P(EC)^m E$ specified by Eq. 11, where h is the step size. Since, the predictor and corrector both have the same order p, Milne's device is applicable and relevant. The following theorem demonstrate adequate condition for the convergence of $P(EC)^m$ or $P(EC)^m E$. In cases where C_{p+1}, C_{p+1} are the computed error constant of the predictor-corrector method, respectively. The following consequence holds.

Proposition: Suppose the predictor method have order p* and the corrector method have order p. Then if $p^* \geq p$ (or $p^* < p$ with $m > p - p)^*$, then the predictor-corrector methods possesses the same order and the same PLTE as the corrector. If ($p^* < p$ and $m = p - p^*$), then the predictor-corrector method possesses the same order as the corrector but different PLTE. If $p^* < p$ and $m \leq p - p^* - 1$ and then the predictor-corrector method possesses the same order equal to $p^* + m$ (thus less than p).

Specifically assume the predictor has order and the corrector has order, p-1 and the corrector has order p, the PEC answers to get a method of order p. Moreover, the $P(EC)^m$ or $P(EC)^m E$ scheme has always the same order and the same PLTE as discussed by Lambert (1973, 1991). Connecting (Faires and Burden, 2012; Lambert, 1973, 1991), Milne's device stated that it is viable to estimate the principal local truncation error of the explicit and implicit (predictor-corrector) method without estimating higher derivatives of Y (x). Assuming that p = p* where p* and p defines the order of the predictor and corrector methods with the same order. Directly, for a method of order p, the principal local truncation errors can be written as:

$$C_{p+1}^{*} h^{(p+1)} y^{(p+1)}(x_n) = y(x_{n+j}) - W_{n+j} + O\left(h^{p+2}\right) \tag{11}$$

Also:

$$C_{p+1} h^{p+1} y^{(p+1)}(x_n) = y(x_{n+j}) - C_{n+j} + O\left(h^{p+2}\right) \tag{12}$$

where, W_{n+j} and C_{n+j} are called the predicted and corrected approximations given by method of order p while C^*_{p+1} and C_{p+1} are independent of h. Neglecting terms of degree and above, it is easy to make estimates of the principal local truncation error of the method as:

$$C_{p+1} h^{p+1} y^{(p+1)}(x_n) = \frac{C_{p+1}}{C_{p+1}^{*} - C_{p+1}} \left| W_{n+j} - C_{n+j} \right| < \varepsilon \tag{13}$$

Noting the fact that $C_{p+1} \neq C_{p+1}$ and $W_{n+j} \neq C_{n+j}$. However, the estimate of the principal local truncation error (Eq. 13) is used to ascertain whether to accept the results of the current step or to reconstruct the step with a smaller step size. The step is accepted based on a test as ordered by Eq. 13 as in Uri and Linda (1998). Equation 13 is the convergence criteria otherwise called Milne's estimate for correcting to convergence. Furthermore, Eq. 13 ensures the convergence criterion of the method during the test evaluation.

CONCLUSION

Investigating the convergence of some selected properties on block predictor-corrector method have been properly analyzed. Block predictor-corrector methods is a compendium of Adams family of the predictor-corrector methods which can be implemented in $P(EC)^m$ or $P(EC)^m$ mode as shown above by Faires and Burden (2012), Lambert (1973) and Uri and Linda (1998). All of these sited above favoured the convergence of block predictor-corrector methods and its implementation for solving nonstiff ODEs. Moreover, the convergence of some selected properties of block predictor-corrector methods possesses the same order, thus, necessitate that the stepnumber of the predictor to be one step greater than the corrector method. Again, the principal local truncation error of both the predictor-corrector methods are considered in the building for the implementation and evaluation of maximum errors. In addition, the implementation is achieved with the support of convergence criteria (stopping criteria). This convergence criteria decide whether the result is accepted or repeated as discussed by Uri and Linda (1998). Finally, the implementation of this method comes with many computational advantages as mention previously by Faires and Burden (2012) and Gear (1971).

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